MTH 2310, LINEAR ALGEBRA
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TEST 2 REVIEW, FALL 2017

- The test will take the full period.
- A calculator/computer will be useful for at least one question on the test, and there will be some questions for which a calculator is not allowed.
- The test will cover sections 1.6-1.9, 2.1-2.4, 3.1, and 3.2.
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapters 1, 2, and 3. In particular:
  - (i) Chapter 1 Supplementary: # 15, 17, 20, 21, and 25
  - (ii) Chapter 2 Supplementary: # 1, 2, 3, 5, 9, 10
  - (iii) Chapter 3 Supplementary: # 1, 2, 3, 5 and 6

  The answers to most of those are in the back of the textbook if you want to check your work.
- You can also look at the old tests on my website. Only certain questions from each test will pertain to our test, though:
  Test 1: #1(c,d only), 5-8 and the extra credit
  Test 2: #2 (i, ii, iii only), 3-9 and the extra credit
- As with the quizzes, it is important that you know not just the answer to a question, but also how to *explain* your answer.
- It is likely that I will do something similar to what I did with the Test 2 on my website, where you have to answer some questions, but will have a choice with the other questions (e.g. you will get top score for 4 of the last 5 five questions).
- Practice problems are listed on the next page.

Some problems to work on in class today (most of these are even-numbered problems from the textbook):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.
   (a) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are not in $\mathbb{R}^2$.
   (b) If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.
   (c) If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.
   (d) If A and B are 3×3 and $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}$, then $AB = \begin{bmatrix} A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3 \end{bmatrix}$.
   (e) If A is invertible, then the inverse of A^{-1} is A itself.
   (f) Let A be a square matrix. If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$.
   (g) If A_1, A_2, B_1 and B_2 are $n \times n$ matrices, $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$, then the product BA is defined but AB is not.
   (h) The determinant of a triangular matrix is the product of the entries on the diagonal.
   (i) If $\det A = 0$, then two rows or two columns are the same, or a row or column is zero.

(2) Determine the value(s) of a such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix} \right\}$ is linearly independent.

(3) Let A be a 3×3 matrix with the property that the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^3$. Explain (without using the IMT) why the transformation must be one-to-one.

(4) Show that if the columns of B are linearly dependent, then so are the columns of AB. (Hint: First, explain why the columns of B being linearly dependent means the same thing as saying that there is a nonzero vector $\mathbf{v}$ such that $B\mathbf{v} = \mathbf{0}$. Now use this to answer the question.) Note: You do NOT know that B is a square matrix, so you cannot use the IMT. (Fun question: Is it true that if the columns of B are linearly independent then so are the columns of AB?)

(5) Find the inverse of $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$.

(6) If L is an $n \times n$ matrix and the equation $L\mathbf{x} = \mathbf{0}$ has the trivial solution, do the columns of L span $\mathbb{R}^n$? Why or why not?

(7) Let $A = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$, where B and C are square. Prove that if A is invertible then B and C must be invertible. (Note: you cannot just invoke the Invertible matrix Theorem, you have to use block matrix multiplication at some point.)

(8) Calculate $\det \begin{bmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix}$.

(9) Use row reduction to find $\det \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}$.

(10) Let U be a square matrix such that $U^T U = I$. Show that either $\det U = 1$ or $\det U = -1$.